Indirect Error Representation using Kanpur Theorem – 1

Anurag Maan[1], Nachiketa Tiwari[2], Prabhat Munshi[3]
Indian Institute of Technology Kanpur
1. amaan@vols.utk.edu; 2. ntiwari@iitk.ac.in(0512-259-6526) 3. pmunshi@iitk.ac.in

Abstract

Computed Tomography is a vital tool in non-destructive imaging and evaluation. Various algorithms have been previously developed based on different mathematical approaches to perform tomographic reconstruction of a volume or a slice of an object under test. It has been established that out of the various algorithms Convolution Back-Projection stands out for its efficiency and speed. Other developments in the field include identification and quantification of an inherent error in tomographic algorithms. This quantification called the Kanpur Theorem-1 can be used to extract information about the nature of reconstructions and can be used to represent error in the reconstructions indirectly. This work attempts at consolidating ways to represent error indirectly using Fractal Dimension computed using the Fractional Brownian motion approach and introduces another parameter as a measure of this indirect error. The analysis show that the inverse of sharpest change in the intensity of the reconstructed image shows healthy agreement with conventionally accepted error representatives like the Fractal Dimension of reconstruction.

----

1. Introduction

The mathematics of reconstructing images from CT (Computed Tomography) scan data is well established.Convolution Back Projection is a popular algorithm to reconstruct images from CT scan data. Since CT scan object can be thought of as a function with finite support, CT images suffer from some inherent error due to certain band-limiting features of the CBP, this issue has been discussed at length and an attempt to quantify this error using certain Radav-Derivatives was made by Munshi et al. [3], a simplified version of this quantification has also been presented by Munshi et al. [2]. This quantification of inherent error indicates a linear relationship between inherent error and the double derivative of the filter function used in reconstruction at Fourier space origin. When this linear relationship was tested for real images (as opposed to Simulated Phantoms where pointwise error is known because the original image is known), the proposed error estimate (inverse of largest gray level in the reconstruction$1/N_{max}$), exhibited a similar linear relationship which indicated that the other errors in a tomographic reconstruction namely instrument and discretization errors are either zero or of the same order as that of inherent error [4]. In the same work [4] Munshi et al. indicated that just like inverse of largest gray level Fractal Dimension of an image which quantifies texture can also be used as an error estimate. The concept of a fractal dimension was first introduced by Mandelbrot [5]. Chen et al. [6] showed that the fractal dimension could be obtained in medical images by the concept of fractional Brownian motion. Later Bhat et al. [7] used Fractal Dimension analysis to show that usual L1 and L2 norms (used as errors) exhibit a linear relationship with fractal dimension computed over a certain range of scale [7], another implication of this work was that CT images can be treated as multi-fractals. This discussion infers that fractal dimension of a CT image over a certain range can be used as an error estimate.Our results indicate that the inverse of sharpest change in the intensity of the reconstructed image shows healthy agreement with conventionally accepted error representatives like the Fractal Dimension of reconstruction and L2 norm.

2. Simulation Parameters

The Convolution Back Projection algorithm was implemented on MATLAB for the test phantoms in figure 1. The test phantoms S1 through S5 were reconstructed using the filters

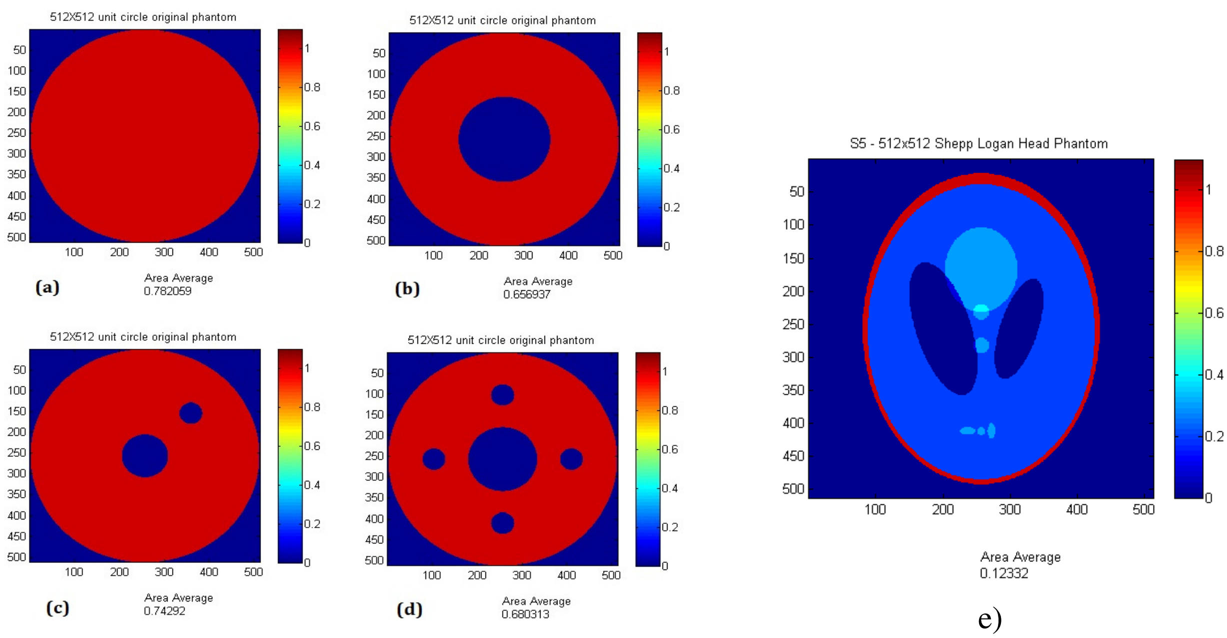

Figure – 1 Test Phantoms a) S1 b) S2 c) S3 d) S4 e) S5

$$W(R) = B + (1-B)[\cos\{\frac{\pi R}{A}\}] \quad (1)$$

$$E = \frac{\left(\sum_{n=0}^{N} (f-\tilde{f})^2\right)^{\frac{1}{2}}}{N} \quad (2)$$

Table - 1		
FILTER CODE	B	*W``(0)*
H99	0.999	0.001
H91	0.917	0.083
H90	0.900	0.100
H80	0.800	0.200
H75	0.750	0.250
H70	0.700	0.300
H60	0.600	0.400
H54	0.540	0.460
H50	0.500	0.500

defined by equation (1) whose parameters are defined in Table 1. For each reconstruction RMSE error (L2 Norm) was calculated using equation (2). Where, f =Original object (Cyber Phantom); $\tilde{f}$ = Reconstructed Object and N = Total Number of pixels.

It has been shown by Munshi [2,3] that simplified form of the inherent error at a given point (r,φ) in the object cross-section, is given by,

$$E_1(r,\phi) = kW''(0)(\nabla^2 f(r,\phi)) \quad (3)$$

Where, $$W''(0) = \frac{\partial^2 W(R)}{\partial R^2} \qquad (4)$$

$\nabla^2 f$ is the Laplacian of f and k is a constant depending on the data ray spacing related to the cut-off frequency R_c. Equation (3) is valid for the objects having certain smoothness properties [3]. The error represents the point-wise theoretical error in reconstruction. This error can be computed for different filters from table-1 and a so called KT – 1 plot can drawn to extract information about the reconstruction, as indicated by eqn (3) the graph is linear and provides information on smoothness, average amplitude and magnitudes of discretization and instrument error [3] in comparison to the inherent error.

3. Error Estimates

Unlike in simulated cyber phantoms,the cross-section physics (distribution)for real objects is unknown; hence the error in reconstruction cannot be calculated directly. This fact motivates an indirect representation of error. It has been reported earlier that [4], for a given data set, sharpness can be used as an indicator of the error behavior, arising due to the choice of the filter function. If the image consists of a single point, then the sharpness parameter corresponds to N_{max}, the maximum grey level (linear absorption coefficient) in the reconstruction [3].Munshi et al. proposed the use of $1/N_{max}$ [4] as an error estimate with the following support, "These numbers are related to the Delta Function Response of the Filters." As mentioned earlier Bhat et al [7] in a separate development have concluded that fractal dimension of an image intuitively relates to roughness of the surface generated by the intensity values of each pixel of the image under consideration. They showed that L1 and L2 errors exhibit a linear relationship with respect to H (Hurst Coefficient) for different filter signatures (W``(0)).

Leading on from this observation it can further argued that a since a tomographic reconstruction is a discrete approximation to a continuous function. Another error estimate may be the inverseof the sharpest change between two neighboring pixels in any reconstruction ($1/dN_{max}$). The sharpest pixel to pixel change in the immediate neighborhood (at distances 1 and $\sqrt{2}$) should be an indicator of the magnitude of frequency components in the neighborhood of R_c in frequency domain; thus implying that the sharpest change in the grey level of any reconstruction should be inversely proportional to E1. This approach is tested and conclusions have been drawn in this work. We now present the KT-1 plots of the cyber phantoms and all of the error estimates introduced so far.

4. Results and Discussion

For simulated objects we are particularly interested in the goodness of fit for the KT-1 plot. It can be argued that since RMSE is a direct error measure, an indirect error estimate must have a comparable goodness of fit as to the RMSE KT-1 plot. Figure – 2 presents the goodness of fit for a KT-1 plot, for all the cyber phantoms at 128x128 pixels, for data collected by 128 rays per projection and 128 projections (parallel beam data collection geometry) for all the error estimates introduced so far. H1 and H2 are Hurst coefficient computed at two different

Normalized scale Ranges (NSR), where NSR isas earlier [6,7]. A complete set of results on all phantoms and the actual values of the Scaled Ranges are presented elsewhere [8].

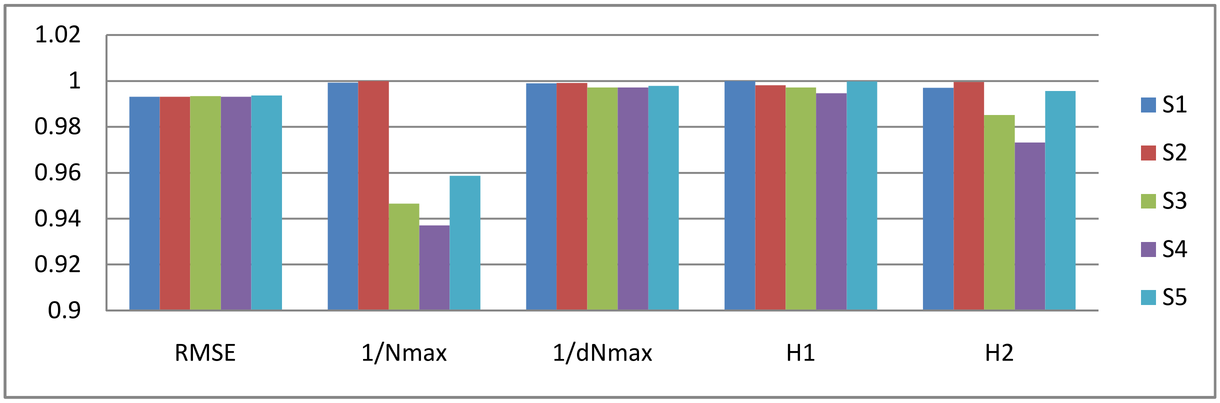

Figure – 2 - Comparison of Goodness of Fit of different error representations from KT-1 plots of S1 through S5 at 128x128

As noise in projection data is introduced the goodness of fit should deteriorate, however, it should be noted that this doesn't indicate the departure from linearity of E_1 w.r.t. $W''(0)$. It just indicates an increase in instrument error. For this purpose White Gaussian Noise (of different magnitude) was added to the projection data of S5 for 256 rays and 256 views. Figure 3 shows the goodness of fit for Signal to Noise ratio (SNR) ranging from 1 dB to 75 dB between 1/Nmax and 1/dNmax.

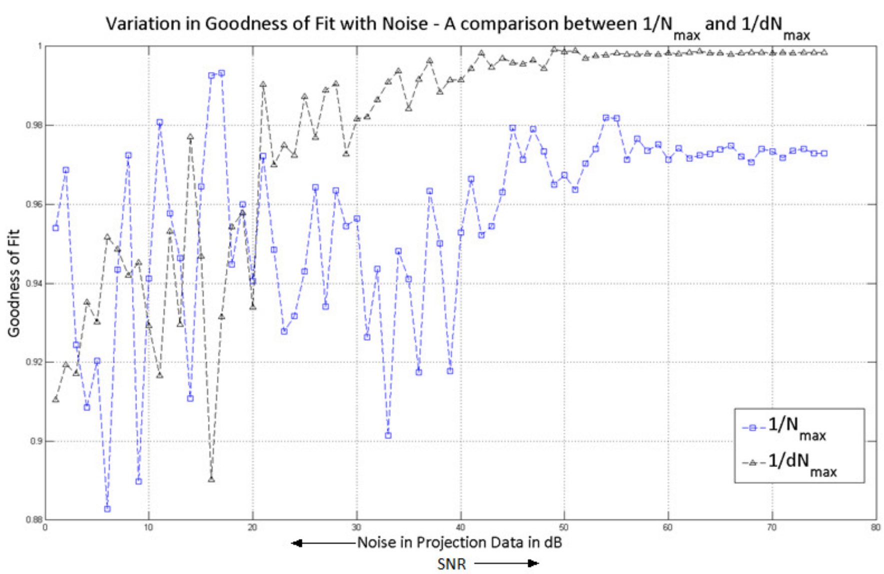

Fig. 3 – S5 – Goodness of Fit with added White Gaussian Noise in Projection Data, a comparison between 1/Nmax and 1/dNmax(2)

$1/dN_{max}$ and the Hurst Coefficient show a goodness of fit which is comparable to RMSE consistently. However, the Fractional Brownian Motion approach to compute Fractal Dimension of an image is computationally intensive. The algorithm in use can be parallelized by domain decomposition to enable its use on High Performance Computing clusters so as to perform the analysis presented in this work for images of larger sizes and to consolidate the findings presented here.

References

[1] G. N. Ramachandran and A. V. Lakshminarayanan, "Three dimensional reconstructions from radiographs and electron micrographs: Application of convolution instead of Fourier transforms," Proc. Nat. Acad. Sci., vol. 68, pp. 2236-2240, 1971.
[2] Munshi P., 1992, Error analysis of Tomographic Filters I: Theory,NDT&E International , 25,pp.191-194.
[3] P Munshi RKS Rathore, KS Ram, MS Kalra,"Error estimates for tomographic inversion", Inverse Problems. 7 (1991), pp 399-408
[4] Munshi P., Maiser, M., Reiter, H., 1997, Experimental aspects of the approximate error formula for tomographic reconstruction, Material Evaluation, 55, pp. 188-191.
[5] Mandelbrot, B. B., 1982, The Fractal Geometry of Nature, Freeman, San Francisco, CA
[6] Chen, C.-C., Deponte, J.S, Fox, M.D., 1989, "Fractal feature analysis and classification in medical imaging," IEEE Trans. Med. Imaging, 8, pp. 133-142.
[7] Bhatt, V., Munshi, P., Bhattacharjee, J. K., 1991, "Application of fractal dimension for nondestructive testing," Materials Evaluation, 91, pp. 1414-1418.
[8] Maan, A., Performance Evaluation Of A Laser Machining Unit Using X-Ray Micro CT Scanner,M. Tech. Thesis, 2014, Indian Institute of Technology Kanpur